

# THE DEVELOPMENT OF UNSTRUCTURED GRID METHODS FOR COMPUTATIONAL AERODYNAMICS

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# MOTIVATION

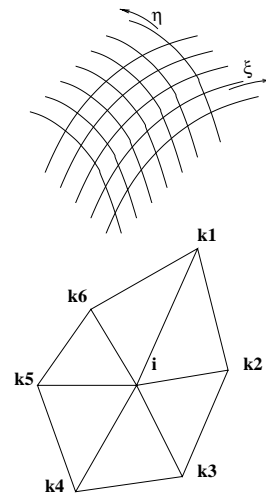
- *Development of Practical Aerodynamic CFD Capability*
  - *Unstructured Grids for Complex Geometries*
  - *Algorithmic Research*
    - \* *Discretization*
    - \* *Solution Techniques*
  - *Computer Science Research*
    - \* *Cache Efficiency*
    - \* *(Vector)/Parallel Processing*
  - *Validation on Realistic Aerodynamic Problems*
    - \* *NASA Wind Tunnel Data*
    - \* *Collaboration with Industry*

# OVERVIEW

- *Unstructured Grid Advantages/Disadvantages*
- *Discretization*
- *Solution Procedures*
  - *Multigrid Methods*
- *Grid Anisotropy*
  - *Directional Preconditioning*
- *Parallelization*
- *Validation*
  - *Large Research Cases on Supercomputers*
  - *Smaller Production Cases on PC Clusters*
- *Current and Future Topics*

# OVERVIEW

- *(Block) Structured Grids*
  - *Logically Rectangular*
  - *Supports Dimensional Splitting Algorithms*
  - *Banded Matrices*
  - *Block Structure for Complex Geometries*
- *Unstructured Grids*
  - *Lists of Cell Connectivity, Graphs (Edges, Vertices)*
  - *Alternate Discretization/Solution Strategies*
  - *Sparse Matrices*
  - *Complex Geometries, Adaptive Meshing*
  - *More Efficient Parallelization (homogeneous)*

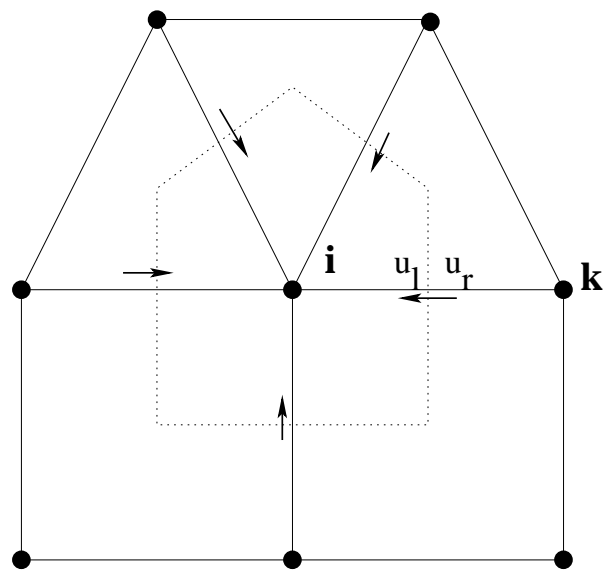


# DISCRETIZATION

- *Governing Equations: Reynolds Averaged Navier-Stokes*
  - *Conservation of Mass Momentum and Energy*
  - *Single Equation Turbulence Model (Spalart-Allmaras)*
    - \* *Convection - Diffusion - Production*
- *Vertex-Based Discretization*
  - *2nd order upwind finite-volume scheme*
  - *6 variables per grid point*
  - *Flow equations fully coupled ( $5 \times 5$ )*
  - *Turbulence equation uncoupled*

# SPATIAL DISCRETIZATION

- *Mixed Element Meshes*
  - *Tetrahedra, Prisms, Pyramids, Hexahedra*
- *Control Volume Based on Median Duals*
  - *Fluxes based on edges*
    - \*  $\mathbf{F}_{ik} = f(\mathbf{u}_{\text{left}}, \mathbf{u}_{\text{right}})$
    - \*  $\mathbf{u}_{\text{left}} = \mathbf{u}_i, \mathbf{u}_{\text{right}} = \mathbf{u}_k$ : *1st order accurate*
    - \*  $\mathbf{u}_{\text{left}} = \mathbf{u}_i + \frac{1}{2} \nabla \mathbf{u}_i \cdot \mathbf{r}_{ik}$
    - \*  $\mathbf{u}_{\text{right}} = \mathbf{u}_k + \frac{1}{2} \nabla \mathbf{u}_k \cdot \mathbf{r}_{ki}$ : *2nd order accurate*
    - \*  $\nabla u_i$  *evaluated as contour integral around CV*
  - *Single Edge Based Data Structure represents all element types*



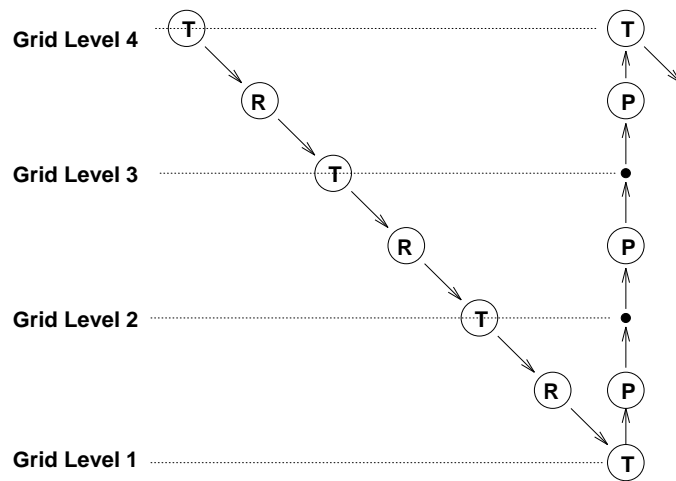
# SOLUTION OF SPATIALLY DISCRETIZED EQUATIONS

$$\frac{du}{dt} + \mathbf{R}(u) = 0$$

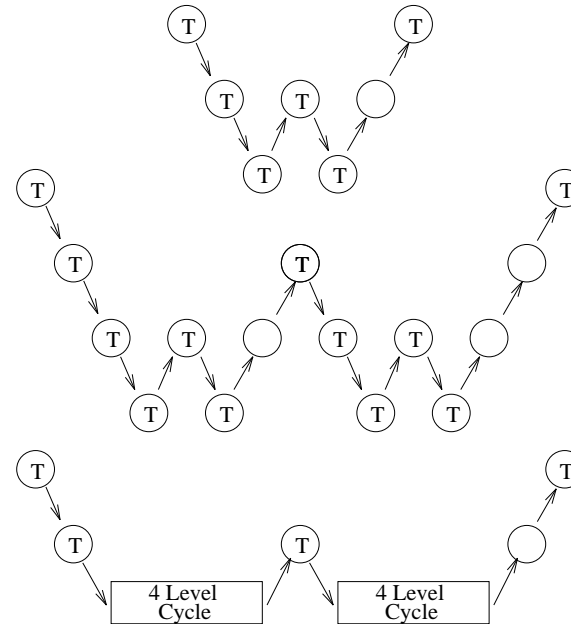
- *Integrate to Steady-State*
- *Explicit :*  $u^{n+1} = u^n - \Delta t \mathbf{R}(u^n)$ 
  - *Simple*
  - *Slow Convergence : Local Procedure*
- *Implicit :*  $(\frac{I}{\Delta t} + \frac{\partial \mathbf{R}}{\partial u})(u^{n+1} - u^n) = -\Delta t \mathbf{R}(u^n)$ 
  - *Large Memory Requirements*
- *Matrix-Free Implicit :*  $\frac{\partial \mathbf{R}}{\partial u} \Delta u = \frac{\mathbf{R}(u) - \mathbf{R}(u + \epsilon \Delta u)}{\epsilon}$ 
  - *Most Effective with Matrix-Based Preconditioner*
- *Multigrid Methods*



# CYCLING STRATEGIES



*V-Cycle*



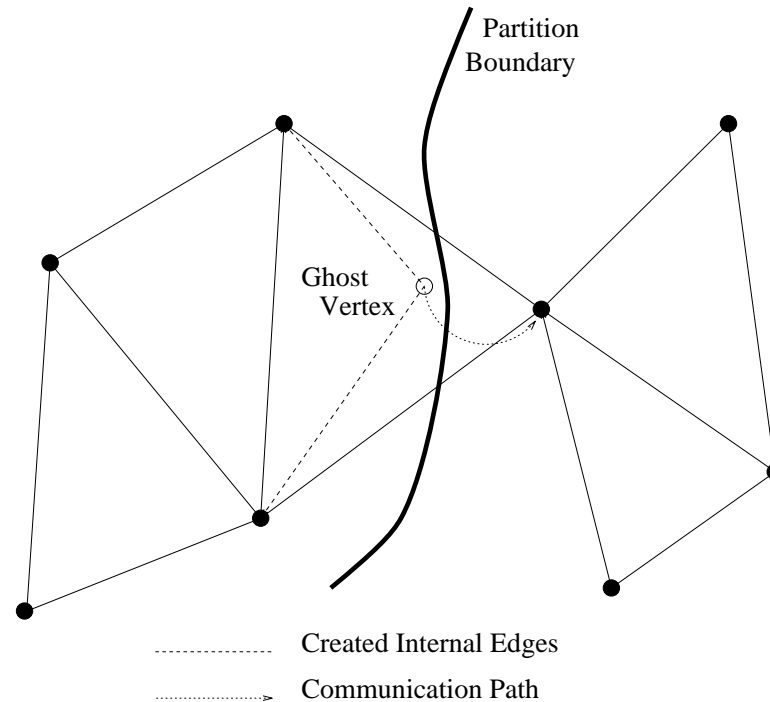
*W-Cycle*

*T = Time-Step*

*R = Restriction*

*P = Prolongation*

# PARALLEL IMPLEMENTATION



- *Intersected Edges Resolved by Ghost Vertices*
- *Generates Communication between Original and Ghost Vertex*
  - *Handled using MPI and/or OpenMP*
  - *Portable, Distributed and Shared Memory Architectures*
- *Local Reordering within partition for Cache-Locality*

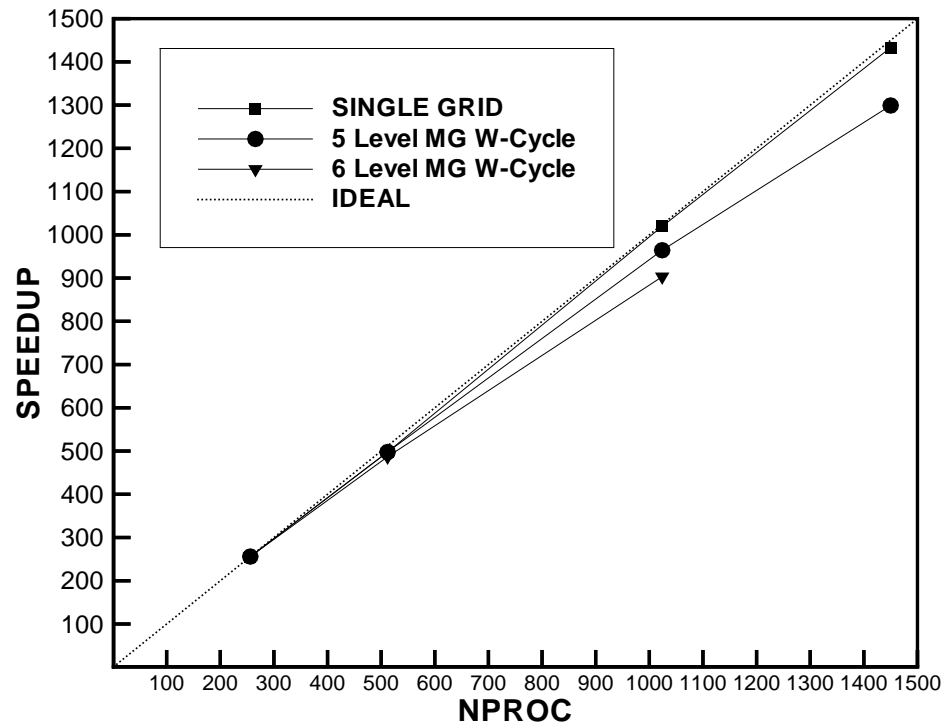
# PARTITIONING

- *Graph Partitioning Must Minimize Number of Cut Edges to Minimize Communication Volume*
- *Standard Graph Based Partitioners: MeTis, CHACO*
  - *Require only Weighted Graph Description of Grid*
    - \* *Edges, Vertices and Weights (taken as unity)*
  - *Ideal for Edge Data Structure*
- *Line Solver Inherently Sequential*
  - *Partition Around Lines using Weighted Graphs*

# SAMPLE CALCULATIONS AND VALIDATION

- *Subsonic High-Lift Case*
  - *Geometrically Complex*
  - *Large Case: 25 million points, 1450 processors*
  - *Research Environment Demonstration Case*
- *Transonic Wing Body*
  - *Smaller Grid Sizes*
  - *Full Matrix of Mach and  $C_L$  conditions*
  - *Typical of Production runs in design environment*

# OBSERVED SPEEDUPS FOR 24.7M PT GRID



## 24.7 Million Pt Case (5 Multigrid Levels)

Platform	No. of Procs	Time/Cyc	Gflop/s
T3E-600	512	28.1	22.0
T3E-1200e	256	38.3	16.1
T3E-1200e	512	19.7	31.4
T3E-1200e	1024	10.1	61.0
T3E-1200e	1450	7.54	82.0

- *Good Multigrid Scalability up to 1450 PEs*
- *Multigrid Scalability Decrease due to Coarse Grid Communication*
  - *(single grid solver not feasible: 100 times slower)*
- *1 hour solution time on 1450 PEs (82 Gflops)*

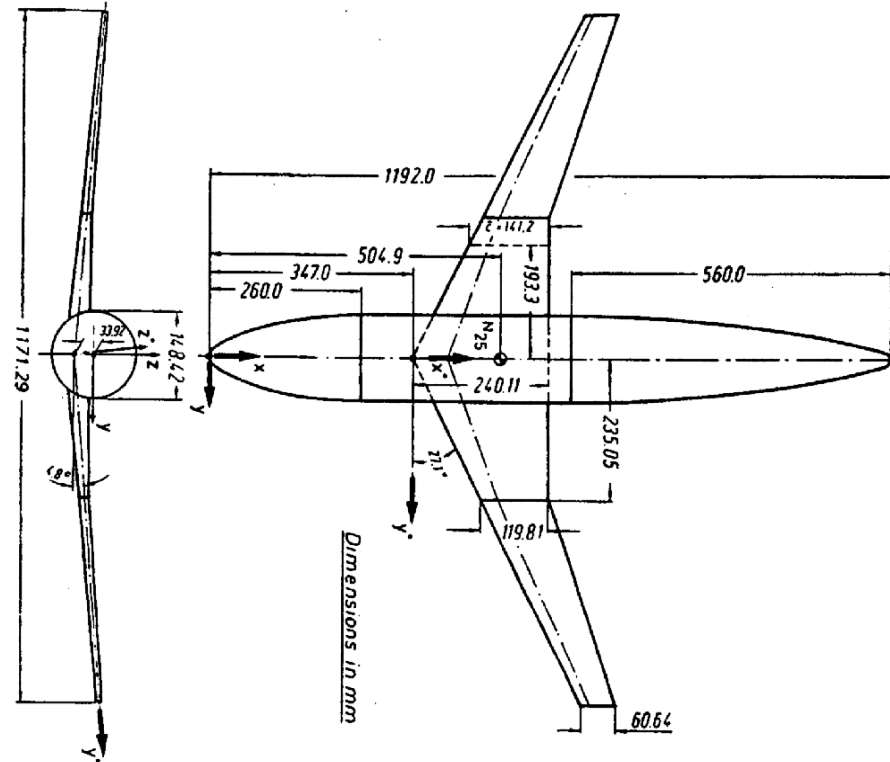
# COMPARISON WITH EXPERIMENTAL DATA

- *Lift versus Incidence Slightly Over Predicted*
- *Drag Polar Well Predicted on Fine Grid*
- *Maximum Lift Point Overpredicted by 1.0 degree*
- *High Lift Flows among most difficult to predict accurately*
  - *Geometric Complexity*
  - *Complex flow physics*
  - *Extremely fine grids required*

# TRANSONIC WING BODY TEST CASE

- *Test Case for AIAA Drag Prediction Workshop*
  - *Assess Capability of Modern CFD Methods for Drag Prediction*
  - *Realistic but Simple Geometry*
  - *Drag polars, Drag Rise Curves*
    - \* *Typical for aircraft design studies*
- *Grid Resolution Effects*
- *Rapid Turnaround for Large Number of Cases on Commodity Hardware*
- *Joint Work with Cessna Aircraft (D. Levy)*

# DRAG PREDICTION WORKSHOP



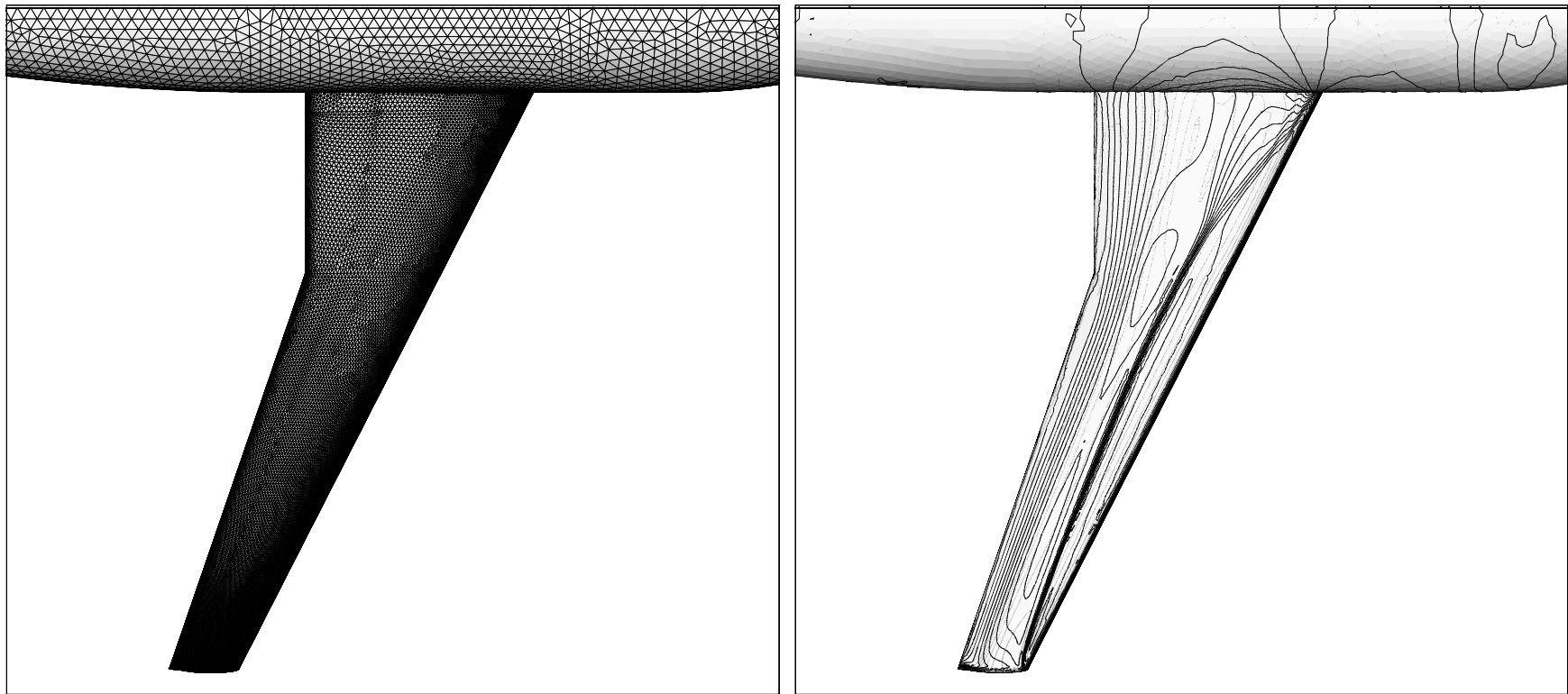
- *DLR-F4 Wing-Body Configuration*
- *Supplied Grid, Custom built Grids*
- *Mandatory Cases:*
  - *Fixed Point  $M=0.75$ ,  $C_L=0.5$ , Drag Polar at  $M=0.75$*
- *Optional Cases*
  - *Drag Rise Curves (Drag vs. Mach at constant  $C_L$ )*



## CASES RUN

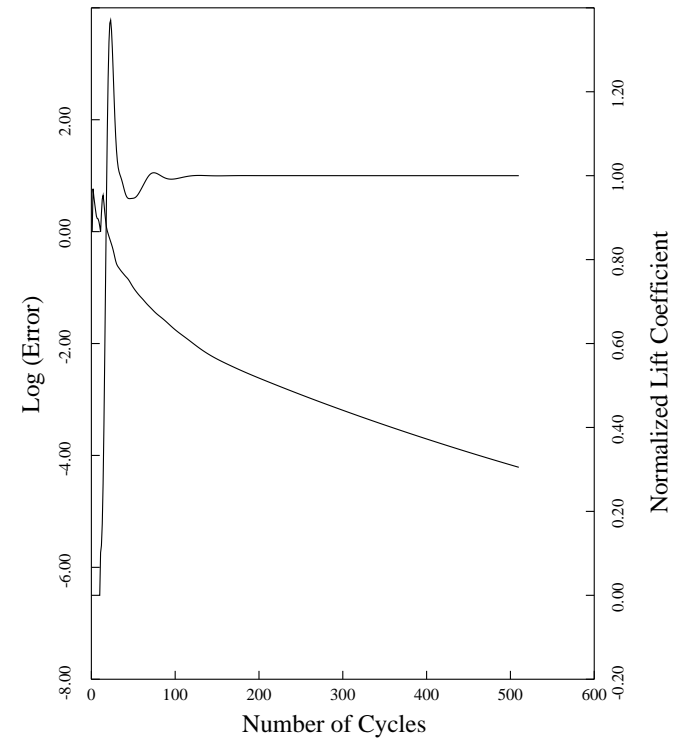
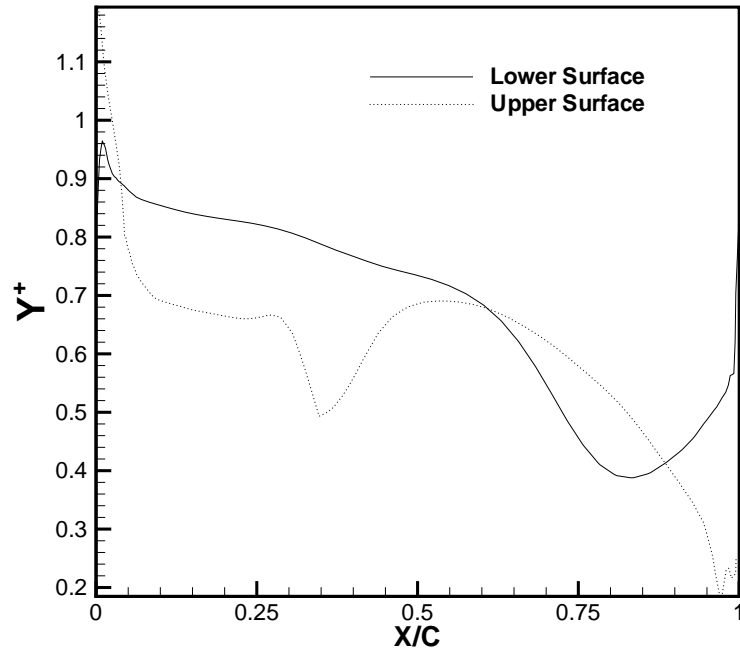
- *BASELINE GRID: 1.6 million points*
  - *Full Drag Polars for Mach Numbers: 0.5, 0.6, 0.7, 0.75, 0.76, 0.77, 0.78, 0.8*
  - *Interpolated Incidence on Polars at Prescribed Lift Value*
  - *Total: 72 cases*
- *MEDIUM GRID: 3.0 million points*
  - *Full Drag Polars for Each Mach Number*
  - *Total: 48 cases*
- *FINE GRID: 13 million points*
  - *Computed Drag Polar at Mach = 0.75*
  - *Computed  $C_L=0.5$  case at Mach=0.75*
  - *Total: 7 cases*
  - *Highest Incidence case not fully converged*

## SAMPLE SOLUTION ON BASELINE GRID (1.65 M pts)



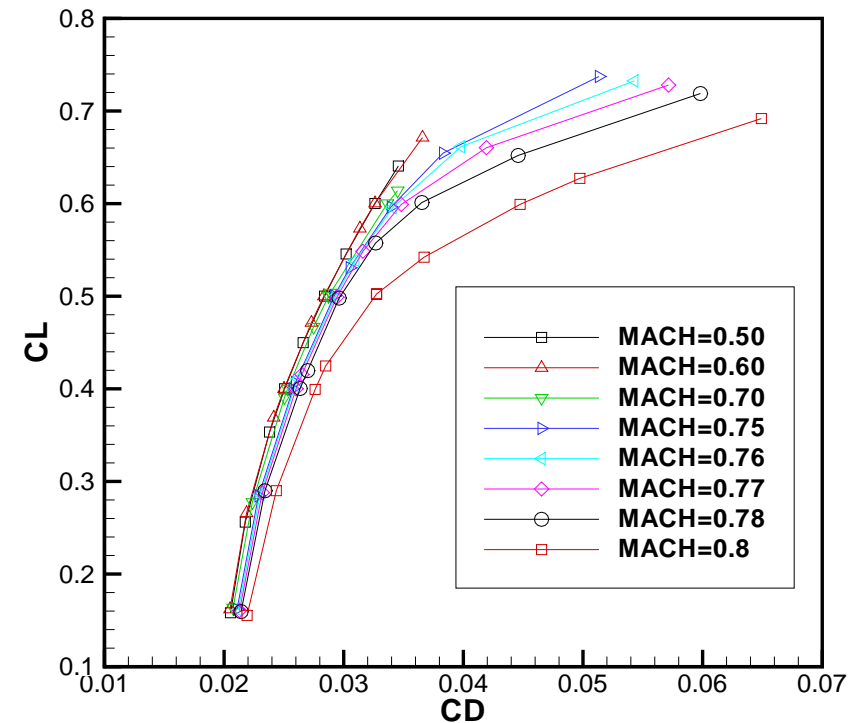
- *Mach = 0.75,  $C_L = 0.6$ ,  $Re = 3$  million*
- *Baseline Grid (1.65 million points)*

# SAMPLE SOLUTION ON BASELINE GRID



- *Adequate Boundary Layer Resolution on Baseline Grid*
- *Force Coefficients Converged in 250 Multigrid Cycles for this case*
- *All Cases run Minimum of 500 Multigrid Cycles*

# BASELINE GRID CASES RUN ON ICASE CLUSTER



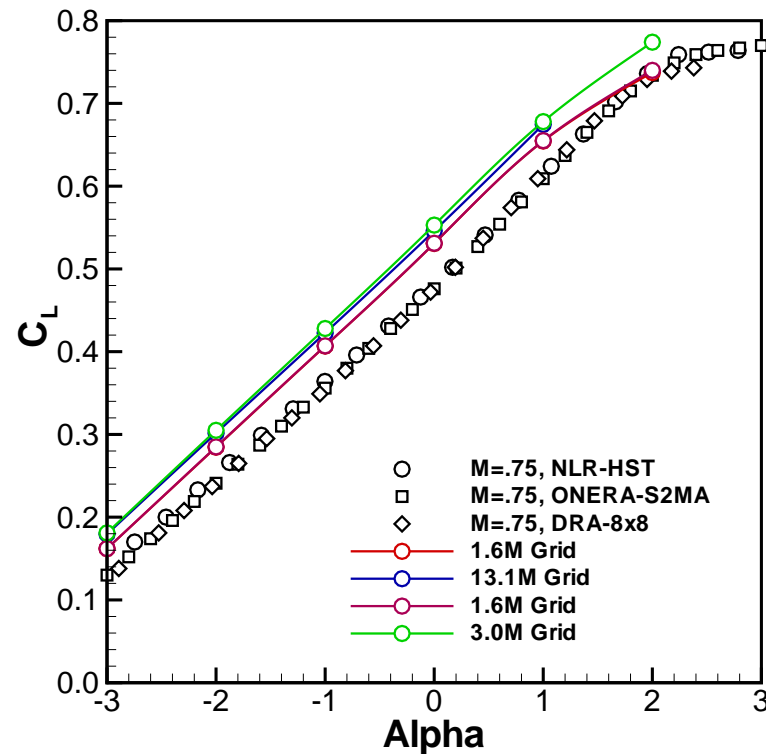
- *Polars for all Mach Numbers: 72 Cases*
- *2.5 hours per case on 16 1.7GHz Pentium CPUs*
- *About 1 week to compute all cases*

# RESULTS FOR CASE 1: Mach = 0.75, CL=0.5, Re = 3M

<i>Case</i>	$C_L$	$\alpha$	$C_D$	$C_M$
<i>Experiment(ONERA)</i>	<i>0.5000</i>	<i>+ .192°</i>	<i>0.02896</i>	<i>-.1301</i>
<i>Experiment(NLR)</i>	<i>0.5000</i>	<i>+ .153°</i>	<i>0.02889</i>	<i>-.1260</i>
<i>Experiment(DRA)</i>	<i>0.5000</i>	<i>+ .179°</i>	<i>0.02793</i>	<i>-.1371</i>
<i>Grid1(1.6Mpts)(ICASE)</i>	<i>0.5004</i>	<i>- .241°</i>	<i>0.02921</i>	<i>-.1549</i>
<i>Grid1(1.6Mpts)(Cessna)</i>	<i>0.4995</i>	<i>- .248°</i>	<i>0.02899</i>	<i>-.1548</i>
<i>Grid2(3.0Mpts)</i>	<i>0.5000</i>	<i>- .417°</i>	<i>0.02857</i>	<i>-.1643</i>
<i>Grid3(13Mpts)</i>	<i>0.5003</i>	<i>- .367°</i>	<i>0.02815</i>	<i>-.1657</i>

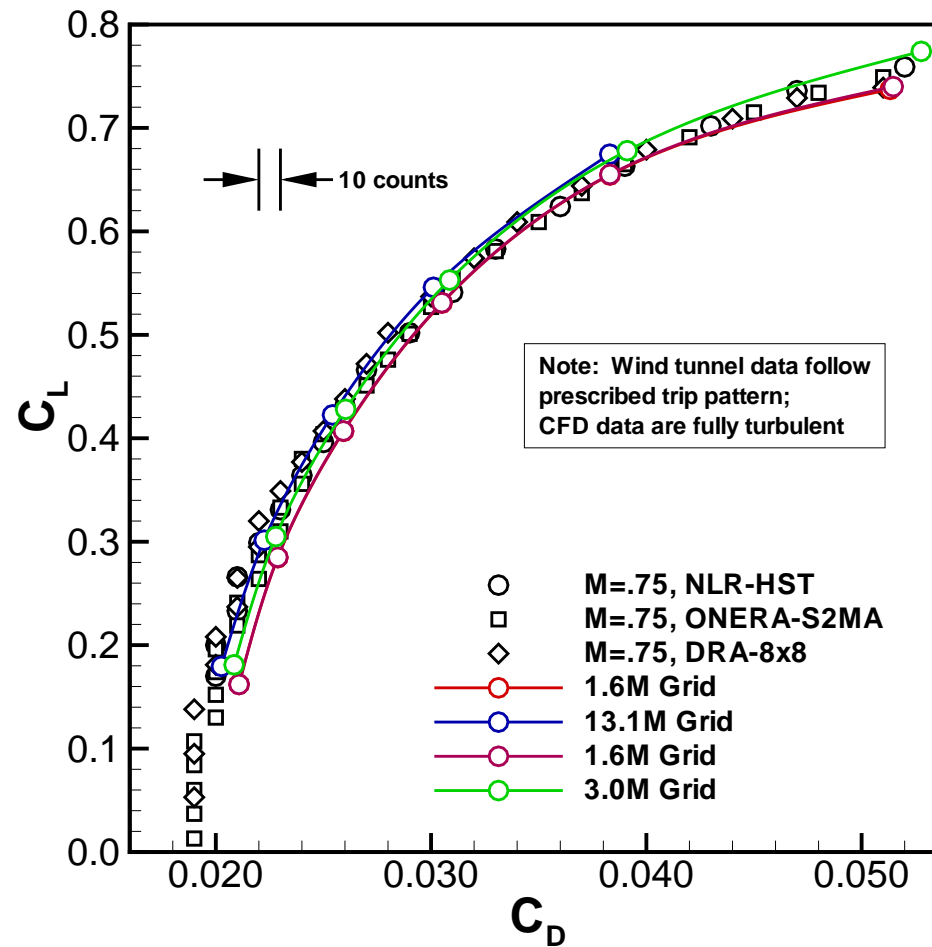
- *Good Overall Drag Agreement (10 counts)*
- *Notable Incidence Offset*

# LIFT VERSUS INCIDENCE



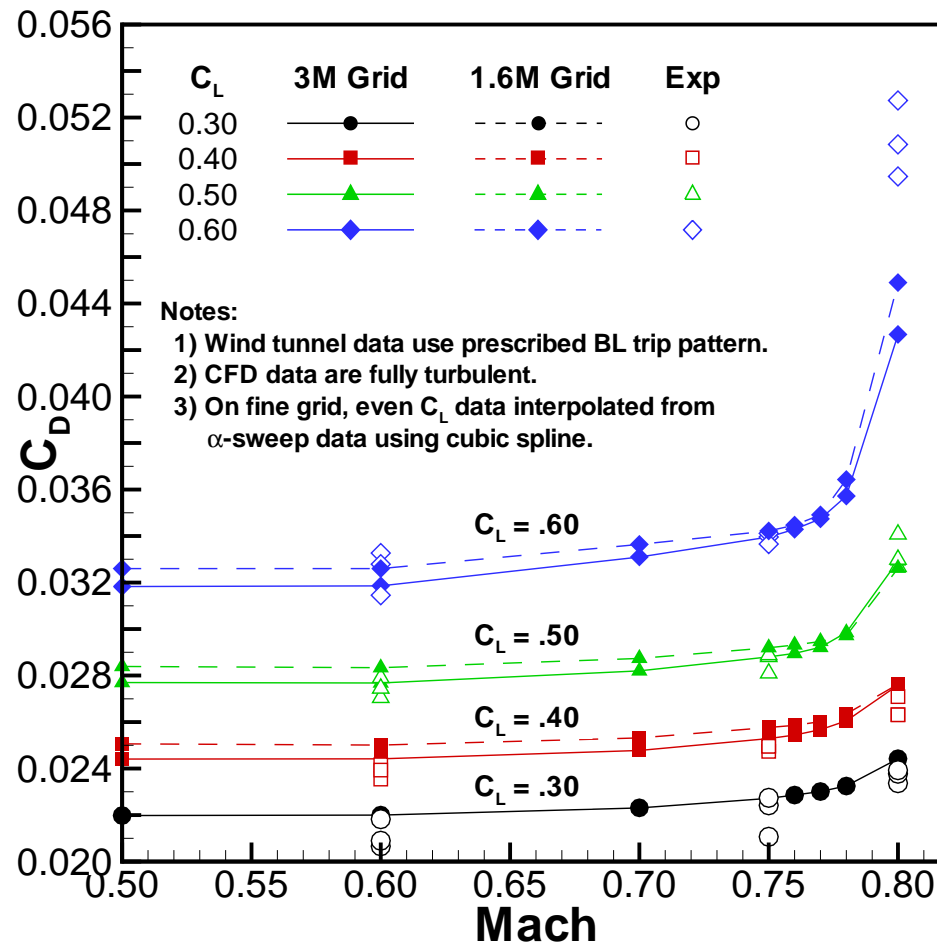
- *Substantial Overprediction of Lift at Given Incidence*
  - *Observed by majority of workshop participants*
- *Slope Overpredicted by  $\approx 5\%$*
- *Unaffected by Grid Resolution*

## DRAG POLAR FOR MACH= 0.75 (CASE 2)



- *Good Drag Prediction Despite  $C_L$  Shift*
- *Better Agreement at Low  $C_L$  with Increased Grid Resolution*

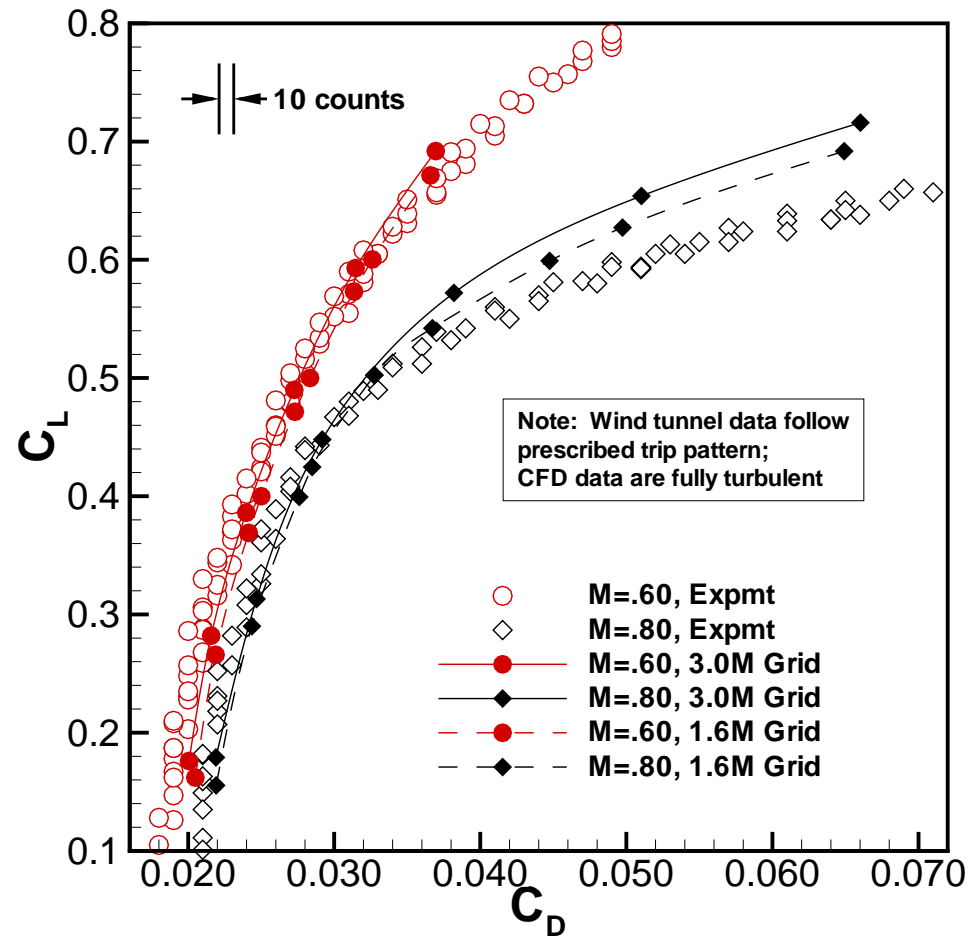
# DRAG RISE COMPARISON WITH EXP. DATA (CASE 4)



- Reasonable Overall Comparison for Relatively Coarse Grid
- Increased Discrepancies at Higher Mach Number and Lift



# ADDITIONAL DRAG POLARS



- *Increased Accuracy for Finer Grid at Lower Lift Values*
- *Increased Discrepancies at Higher Mach Number and Lift*

# DRAG UNDERPREDICTION AT HIGH $C_L$ /Mach

- *Separation Likely Underpredicted at High  $C_L$ /Mach Conditions*
  - *Influence of Turbulence Models*
- *Free Transition in Computations*
  - *Computationally observed  $\approx 5\%$  to  $7\%$  chord*
  - *Experimentally Set  $15\%$ (upper) and  $25\%$ (lower) chord*
- *Possible Effects due to  $C_L$ -Incidence Offset*

## VALIDATION SUMMARY

- *Unstructured Grid Methods Comparable and Often Superior to Structured Counterparts*
  - *Similar Accuracy*
  - *Reduced Setup Time*
  - *Good Parallelization Characteristics*
- *CFD Methods Perform Well at Design Conditions (Attached Flow)*
- *High Incidence, High Lift More Problematic*
- *Transition, Turbulence Modeling Important Issues*
- *Grid Resolution always an Issue*

# CURRENT AND FUTURE RESEARCH AREAS

- *Adaptive Meshing*
  - *Mixed Element Subdivision*
  - *Refinement Criteria Pacing Issue*
  - *Dynamic Load Balancing for Parallel Computing*
- *Unsteady Flows*
  - *Implicit Time Solution Procedures*
  - *Moving Grids, Overlapping Grids*
  - *Overlapping Grids*
- *LES and DES Simulations of Separated Flows*
- *Higher-Order Methods*
  - *4th order in Time (Implicit Runge-Kutta)*
  - *Discontinuous Galerkin, SUPG Methods*